

Ques ① state Quotient law.

Sol The quotient law of exponents states that to divide two powers with same base $\frac{a^m}{a^n} = a^{m-n}$

The quotient law of tensors states that if the inner product of an unknown set of components with an arbitrary tensor $A_{ij} B_j = C_i$ of the inner product of set of fun. with an arbitrary vector is a tensor, then the set of fun. is also tensor.

Ques ② Show that δ_j^i is a mixed tensor of rank two.

Sol - The Kronecker delta is defined as

$$\delta_j^i = \begin{cases} 1 & i=j \\ 0 & i \neq j \end{cases} \text{ in any co-ordinate system } x^i$$

we have to show δ_j^i transforms like a mixed tensor of type (1,1)

$$T_{j'}^{i'} = \frac{\partial x^{i'}}{\partial x^k} \frac{\partial x^j}{\partial x^{j'}} T_j^k$$

R.H.S δ_l^k is $\delta_j^k A^l = A^k$ Here it contracts k with l .

$$= \frac{\partial x^{i'}}{\partial x^k} \frac{\partial x^k}{\partial x^{j'}} \therefore \delta_l^k \text{ replace } l \text{ by } k.$$

By chain rule of partial derivatives

$$\frac{\partial x^{i'}}{\partial x^k} \frac{\partial x^k}{\partial x^{j'}} = \frac{\partial x^{i'}}{\partial x^{j'}} \text{ But } \frac{\partial x^{i'}}{\partial x^{j'}} = \delta_j^{i'}$$

$$\Rightarrow \delta_{j'}^{i'} = \frac{\partial x^{i'}}{\partial x^k} \frac{\partial x^k}{\partial x^{j'}} \delta_l^k \Rightarrow \text{mixed tensor of rank 2.}$$

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Ques ③ what do you mean by contraction of a tensor.

Sol - Contraction is an operation by which the rank of a mixed tensor is reduced by 2.

Def If in a mixed tensor, one contravariant index and one covariant index are set equal to each other and summed over according to Einstein summation convention the result is a tensor of rank $(r-1+s+t-1)$
 $= r+s-2$.

Ques ④ Contract the Kronecker delta.

Sol It's a mixed tensor of rank 2 denoted by δ_j^i

contraction means put $i=j$ and sum.

$$\text{Contraction of } \delta_j^i = \delta_i^i = \sum_{i=1}^n \delta_i^i$$

$$\delta_i^i = 1+1+1 + \dots + 1 (n \text{ times}) = n.$$

Here n dimension of the space.

Ex $\delta_j^i A^j = A^i$, $\delta_j^i A_i = A_j$

Ques ⑤ Define Darboux vector, write Serret-Frenet formula in terms of Darboux vector.

Sol - let C be a space curve with arc lengths, unit tangent t , principal normal n and binormal b , curvature k and torsion τ .

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Then the vector $w = \tau t + kb$ is called Darboux Vector.

Serret-Frenet Formula in terms of Darboux vector.

Serret-Frenet Formula $t' = kn$, $n' = -kt + \tau b$.

$$b' = -\tau n$$

in terms of Darboux vector w

$$t' = w \times t \quad n' = w \times n \quad b' = w \times b.$$

Ques 6 Find k for the space curve $x = a \cos t$, $y = a \sin t$, $z = at \cos \alpha$.

Sol - $r(t) = (a \cos t, a \sin t, at \cos \alpha)$

$$r' = (-a \sin t, a \cos t, a \cos \alpha)$$

$$|r'| = a\sqrt{1} \quad r'' = (-a \cos t, -a \sin t, 0)$$

$$r' \times r'' = \begin{vmatrix} i & j & k \\ -a \sin t & a \cos t & a \cos \alpha \\ -a \cos t & -a \sin t & 0 \end{vmatrix} \Rightarrow |r' \times r''| = \sqrt{a^4 \cos^2 \alpha \sin^2 t + a^4 \cos^2 \alpha \cos^2 t + 0}$$

$$\text{Curvature formula } k = \frac{|r' \times r''|}{|r'|^3} = \frac{1}{a} \sin^2 \alpha.$$

$\Rightarrow$ It is a circular helix.

Ques 7 Define rectifying plane and write down the eq. of rectifying plane.

Sol - The plane which contains the tangent t and binormal b at a point P of a curve and is orthogonal to the principal Normal n is called the rectifying plane.

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If (x, y, z) is point on plane and (x_0, y_0, z_0) is point on curve.

$$(x-x_0) N_x + (y-y_0) N_y + (z-z_0) N_z = 0$$

where $N = r' \times (r'' \times r')$ is principal normal direction.

Ques ⑧ Define the edge of regression.

Sol - Let S be a developable surface generated by a one-parameter family of planes

$F(x, y, z, t) = 0$ where t is a parameter.

The characteristic of S is the line of intersection of two consecutive generators $F=0$ and $F_1=0$

This locus is a space curve which is called the edge of regression

Ques ⑨ Define lines of curvature and write down the differential eq. of lines of curvature.

Sol Lines of curvature - A curve on a surface whose tangent at every pt. is in the principal direction at that point is called a line of curvature

Let Surface be $s = r(u, v)$

First & Second Fundamental Coefficients are

E, F, G, L, M, N

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then $dz = r_u du + r_v dv$

$dN = N_u du + N_v dv$

condition for principal direction is $dN + K_n dz = 0$

Eliminate K_n we get $\frac{M du + N dv}{F du + G dv}$

Therefore differential eq. is

$(E du + F dv)(M du + N dv) - (F du + G dv)(du + M dv)$

$\Rightarrow (EM - FL) du^2 + (EN - GL) du dv + (FN - G^2) dv^2 = 0$

Ques (10) Prove that $[ij, k] + [jk, i] = \frac{\partial g_{jk}}{\partial x^i}$

Sol - It's a Christoffel symbol of 1st kind.

$[ij, k] = \frac{1}{2} \left(\frac{\partial g_{jk}}{\partial x^i} + \frac{\partial g_{ik}}{\partial x^j} - \frac{\partial g_{ij}}{\partial x^k} \right)$

where g_{ij} is the metric tensor $g_{ij} = g_{ji}$

$[ij, k] = \frac{1}{2} (\partial_i g_{jk} + \partial_j g_{ik} - \partial_k g_{ij}) \quad \text{--- (1)}$

where $\partial_i = \frac{\partial}{\partial x^i}$ Interchange $i \leftrightarrow j$ and $k \leftrightarrow i$

$[jk, i] = \frac{1}{2} \left(\frac{\partial g_{ki}}{\partial x^j} + \frac{\partial g_{ji}}{\partial x^k} - \frac{\partial g_{jk}}{\partial x^i} \right)$

$[jk, i] = \frac{1}{2} (\partial_j g_{ki} + \partial_k g_{ji} - \partial_i g_{jk}) \quad \text{--- (2)}$

Add eq. (1) & (2) $\partial_i g_{jk} - \partial_i g_{jk} = 0$

$\partial_k g_{ji} - \partial_k g_{ij} = 0$

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we have $[ij, k] + [jk, i] = \frac{1}{2} \left[\frac{\partial}{\partial x^j} g_{ik} + \frac{\partial}{\partial x^k} g_{ij} \right]$

$$\Rightarrow [ij, k] + [jk, i] = \frac{\partial g_{ik}}{\partial x^j} \quad \text{Hence proved}$$

Ques (11) Prove that contract tensor A_j^i is a scalar.

Sol It's a mixed tensor of type (1,1) $A_j^i = \frac{\partial x'^i}{\partial x^p} \frac{\partial x^q}{\partial x'^j} A_q^p$ where summation over $p, q = 1$ to n .

Contract set $i=j$ and sum. Let $i=j=k$ in eq. and sum over k .

$$A_k^k = \frac{\partial x'^k}{\partial x^p} \frac{\partial x^q}{\partial x'^k} A_q^p \quad \text{interchange order} = \frac{\partial x'^k}{\partial x^p} \frac{\partial x^p}{\partial x'^k} A_k^k$$

But the chain rule $\frac{\partial x'^k}{\partial x^p} \frac{\partial x^p}{\partial x'^k} = \frac{\partial x^q}{\partial x^p} = \delta_p^q$ is Kronecker delta.

Ques (12) Show that $g_{ij} dx^i dx^j$ is an invariant.

Sol $ds^2 = g_{ij} dx^i dx^j$ is the square of the distance

between two nbd points T, S - $g'_{kl} dx'^k dx'^l = g_{ij} dx^i dx^j$

we know $g'_{kl} = g_{pq} \frac{\partial x^p}{\partial x'^k} \frac{\partial x^q}{\partial x'^l}$ — (1), $dx'^k = \frac{\partial x'^k}{\partial x^i} dx^i$ — (2)

$$dx'^l = \frac{\partial x'^l}{\partial x^j} dx^j \quad \text{— (3) In } ds^2 = g'_{kl} dx'^k dx'^l \text{ Put (1) (2) \& (3) we get.}$$

$$= g_{pq} \left(\frac{\partial x^p}{\partial x'^k} \frac{\partial x^q}{\partial x'^l} \right) \left(\frac{\partial x'^k}{\partial x^i} \frac{\partial x'^l}{\partial x^j} \right) dx^i dx^j \quad \text{Hence } ds^2 = g_{pq} \delta_i^p \delta_j^q dx^i dx^j$$

Ques (13) Find the Gaussian curvature at any pt. of a sphere of radius a .

Sol. Let sphere $x^2 + y^2 + z^2 = a^2$

Parametrization by u, v

$$r(u, v) = (a \sin u \cos v, a \sin u \sin v, a \cos u)$$

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Gaussian curvature $k = \frac{LN - M^2}{EG - F^2}$ using inward normal to make $k > 0$

$$k = \frac{a \cdot a \sin^2 u - 0}{a^2 \cdot a^2 \sin^2 u - 0} = \frac{a^2 \sin^2 u}{a^4 \sin^2 u} = \frac{1}{a^2}$$

with outward Normal $L = -a, N = -a \sin^2 u$

Product $LN = a^2 \sin^2 u \Rightarrow k = \frac{1}{a^2}$ constant (similarly)

Thus at every point of sphere Gaussian curvature is same and equals reciprocal of square of

radius. $H = \frac{EN + GL - 2FM}{2(EG - F^2)} = \frac{1}{a}$ (with inward Normal)

Hence sphere is totally umbilic both principal

curvatures $k_1 = k_2 = \frac{1}{a}, K = k_1 k_2 = \frac{1}{a^2}$

Ques (14) Find the equation of osculating plane at any point t on the helix $x = a \cos t, y = a \sin t, z = ct$

Sol - At the current point on the helix let

$$r(t) = (a \cos t, a \sin t, ct)$$

Osculating plane is the plane containing t and n .

It's normal is $\parallel r' \times r''$ Here $r' = (-a \sin t, a \cos t, c)$

$$r'' = (-a \cos t, -a \sin t, 0)$$

$$r' \times r'' = \begin{vmatrix} x & y & z \\ -a \sin t & a \cos t & c \\ -a \cos t & -a \sin t & 0 \end{vmatrix} = a \begin{vmatrix} x - a \cos t & y - a \sin t & z - ct \\ -a \sin t & a \cos t & c \\ -a \cos t & -a \sin t & 0 \end{vmatrix}$$

Expand the determinant

$$(x - a \cos t)(a \cos t) - (y - a \sin t)(a \cos t) + (z - ct)a^2$$

$$a c \sin t (x - a \cos t) - a c \cos t (y - a \sin t) + a^2$$

$$c \sin t \cdot x - c \cos t \cdot y + az = act$$

So the eq. of the osculating plane to the helix at any point t is $-c \sin t \cdot X - c \cos t \cdot Y + aZ = act$ where (X, Y, Z) represent the coordinates of a general point on the plane. The +ve vector $\vec{r}(t)$ of any point on the helix is given by $\vec{r}(t) = (a \cos t)\hat{i} + (a \sin t)\hat{j} + ct\hat{k}$. We need the velocity vector $\vec{r}'(t)$ and acceleration vector $\vec{r}''(t)$. $\vec{r}'(t) = -a \sin t \hat{i} + a \cos t \hat{j} + c \hat{k}$, second derivative $\vec{r}''(t) = -a \cos t \hat{i} - a \sin t \hat{j}$.

Ques (15) Define tangent, normal and binormal of the space curve.

Sol - The tangent indicates the instantaneous direction of motion along the curve we find it by diff. the position vector and normalizing it to a length of 1. $T(t) = \frac{\vec{r}'(t)}{\|\vec{r}'(t)\|}$

T has a constant magnitude of 1, its derivative $T'(t)$ is always orthogonal to T . Normalizing this derivative gives the unit normal vector

$$N(t) = \frac{T'(t)}{\|T'(t)\|}$$

The vector cross product of the unit tangent and unit normal vectors $B(t) = T(t) \times N(t)$.

Ques (16) what do you mean by first fundamental form or metric surface.

Sol - For a surface $\vec{r}(u, v)$ the first fundamental form is expressed as the square of the arc length element $ds^2 = Edu^2 + 2Fdu dv + Gdv^2$

In tensor analysis and differential geometry, the metric surface is a fundamental geometric object used to measure distances, angles and volumes on a curved surface. It's denoted by g_{ij} or g^{ij} (Covariant, contravariant). A metric surface uses the metric tensor to find out arc length element (ds) on a curved surface.

$$ds^2 = g_{ij} dx^i dx^j$$

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Ques (17) Show that the curve $u+v = \text{constant}$ is a geodesics on a surface with metric.

Sol- Given family of curves $u+v=c$ can form a geodesic. Differentiate $u+v=c$ with respect to the parameter t .

$$\dot{u} + \dot{v} = 0 \Rightarrow \dot{v} = -\dot{u} \quad \text{diff again second time } \ddot{u} + \ddot{v} = 0 \Rightarrow \ddot{v} = -\ddot{u}$$

The curve $u+v = \text{constant}$ are geodesics on a surface with the metric $ds^2 = (1+u^2)du^2 - 2uv du dv + (1+v^2)dv^2$. Now we use Euler Lagrange equations to check a curve is a geodesic.

A curve is a geodesic if it satisfies the system of Euler-Lagrange.

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{u}} \right) - \frac{\partial L}{\partial u} = 0 \quad \text{and} \quad \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{v}} \right) - \frac{\partial L}{\partial v} = 0$$

$$\text{Now } \frac{\partial L}{\partial \dot{u}} = 2(1+u^2)\dot{u} - 2uv\dot{v}, \quad \frac{\partial L}{\partial u} = 2u\dot{u}^2 - 2v\dot{u}\dot{v} \quad (\text{Now put } \dot{v} = -\dot{u})$$

$$\frac{\partial L}{\partial \dot{u}} = 2(1+u^2)\dot{u} - 2uv(-\dot{u}) = 2\dot{u}(1+u^2+uv) \quad \left\{ \begin{array}{l} \frac{d}{dt} [2\dot{u}(1+u^2+uv)] = 2\dot{u}(1+u^2+uv) + 2u\dot{u} \\ \text{Since } \dot{u} = -\dot{u} \Rightarrow 2\dot{u}(\dot{u}(1+u^2+uv)) \Rightarrow \end{array} \right.$$

$$\frac{\partial L}{\partial u} = 2u\dot{u}^2 - 2v\dot{u}(-\dot{u}) = 2\dot{u}^2(u+v)$$

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{u}} \right) - \frac{\partial L}{\partial u} \Rightarrow 2\dot{u}(1+u^2+uv) = 0$$

Ques (18) Find the curvature of the curve given by $r = \{a(3u-u^3), 3au^2, a(3u+u^3)\}$

Sol- Differentiate each component with respect to u .

$$r'(u) = \{a(3-3u^2), 6au, a(3u^2+3)\} \Rightarrow |r'(u)| = 3a\sqrt{2u^4+4u^2+2} = 3a\sqrt{2(u^2+1)^2}$$

$$r''(u) = \{3a(-2u, 2, 2u)\} = 6a(-u, 1, u)$$

$$r'(u) \times r''(u) = (3a)(6a) \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1-u^2 & 2u & u^2+1 \\ -u & 1 & u \end{vmatrix} \Rightarrow r'(u) \times r''(u) = 18a^2 \begin{pmatrix} u^2+1-2u \\ u^2+1 \\ -u^2+1 \end{pmatrix}$$

$$|r'(u) \times r''(u)| = 18a^2 \sqrt{2(u^2+1)^2} = 18\sqrt{2} a^2 (u^2+1)$$

$$k(u) = \frac{|r'(u) \times r''(u)|}{|r'(u)|^3} \Rightarrow k(u) = \frac{1}{3a(u^2+1)^2}$$

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Ques (19) Define Osculating plane.

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Sol. Let P be a point on a space curve, $\vec{r} = \vec{r}(t)$ of class ≥ 2 with $\kappa \neq 0$.

The osculating plane at P , P is the plane having the closest possible contact with the curve at P .

OR The osculating plane is the unique plane that has the closest possible contact with a space curve $\vec{r}(t)$ at a specific Pt. P . For a curve of class ≥ 2 with a non-zero curvature ($\kappa \neq 0$). It is the plane that best fits the curve locally at that point.

It's a limiting position of a plane passing through three distinct Pt. on the curve as these Pt. approach P .

Ques (20) Explain principal Normal and Binormal.

Sol - The principal Normal and Binormal vectors are fundamental components of the Frenet-Serret frame, which is a moving co-ordinate system used in differential geometry to describe the behavior of a 3D space curve at any given point.

Principal Normal Vector ($\vec{N}$)

Ques (21) It measure how sharply the curve is bending and points directly toward the center of curvature

$$\vec{N} = \frac{1}{\kappa} \frac{d\vec{T}}{ds}$$

Binormal - The binormal is a unit vector that is $\perp$ to both tangent and principal Normal vector

$$\vec{B} = \vec{T} \times \vec{N}$$

Ques (21) Contravariant vectors of rank two.

Sol - Let A^{ij} , be n^2 fun. of the co-ordinates $x^1, x^2, \dots, x^n$ of the corresponding fun. $\bar{A}^{ij}$ of the co-ordinates $\bar{x}^1, \bar{x}^2, \dots, \bar{x}^n$ transform like

$$\bar{A}^{ij} = \frac{\partial \bar{x}^i}{\partial x^k} \frac{\partial \bar{x}^j}{\partial x^l} A^{kl}$$

Here A^{ij} are called the components of a contravariant tensor of the rank-two.

Ques: (22) If A_{ij} is a skew-symmetric tensor, prove that $(\delta_j^i \delta_l^k + \delta_l^i \delta_j^k) A_{ik} = 0$.

Sol- Since A_{ij} is skew-symmetric, we have $A_{ij} = -A_{ji}$ for all i, j .

$$\begin{aligned} \text{Now } (\delta_j^i \delta_l^k + \delta_l^i \delta_j^k) A_{ik} &= \delta_j^i (\delta_l^k A_{ik}) + \delta_l^i (\delta_j^k A_{ik}) \\ &= \delta_j^i A_{il} + \delta_l^i A_{ij} = A_{jl} + A_{lj} \quad (\text{using skew symmetric}) \\ &= A_{jl} - A_{jl} = 0. \end{aligned}$$

Ques (23) Prove that the contraction of a mixed tensor A_j^i is a scalar invariant.

Sol- By the law of transformations of tensors

$$\bar{A}_j^i = \frac{\partial \bar{x}^i}{\partial x^k} \cdot \frac{\partial x^l}{\partial \bar{x}^j} A_l^k$$

setting $j=i$ and then summing over i , we obtain $\bar{A}_i^i = \frac{\partial \bar{x}^i}{\partial x^k} \cdot \frac{\partial x^l}{\partial \bar{x}^i} A_l^k = \frac{\partial x^l}{\partial x^k} A_l^k = \delta_k^l A_l^k = A_k^k$

Thus $\bar{A}_i^i = A_k^k$ changing the dummy index k by i we have

$$\bar{A}_i^i = A_i^i \Rightarrow A_i^i \text{ is an invariant}$$

But A_i^i is the contraction of the mixed tensor A_j^i .

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Ques 24) Prove that if L, M, N vanish at all points of a surface then the surface is a plane.

Sol - we know the coefficients of second fundamental form

$$L = \vec{r}_{uu} \cdot \vec{n}, \quad M = \vec{r}_{uv} \cdot \vec{n}, \quad N = \vec{r}_{vv} \cdot \vec{n}$$

Given $L=0 \Rightarrow M=0, N=0$ at all points on the surface.

$$L = \vec{r}_{uu} \cdot \vec{n} = -\vec{r}_u \cdot \vec{n}_u, \quad M = \vec{r}_{uv} \cdot \vec{n} = -\vec{r}_u \cdot \vec{n}_v = -\vec{r}_v \cdot \vec{n}_u$$

$$N = \vec{r}_{vv} \cdot \vec{n} = -\vec{r}_v \cdot \vec{n}_v, \quad L=0, M=0, N=0$$

$$\vec{n}_u \cdot \vec{r}_u = 0 \quad \text{and} \quad \vec{n}_v \cdot \vec{r}_v = 0 \Rightarrow \vec{n}_v = 0 \text{ if } \vec{n}_u = 0$$

Since $\vec{n}$ is a unit vector $\vec{n} \cdot \vec{n} = 1$

$$\vec{n}_u \cdot \vec{n} = 0 \quad \text{and} \quad \vec{n}_v \cdot \vec{n} = 0$$

$$d(\vec{r} \cdot \vec{c}) = d\vec{r} \cdot \vec{c} + \vec{r} \cdot d\vec{c} \quad \text{since } \vec{c} \text{ is a constant vector } d\vec{c} = 0.$$

$$\Rightarrow d\vec{r} = \vec{r}_u du + \vec{r}_v dv$$

$$d(\vec{r} \cdot \vec{c}) = (\vec{r}_u du + \vec{r}_v dv) \cdot \vec{c} + 0 = 0 + 0 = 0$$

Integrate $d(\vec{r} \cdot \vec{c}) = 0 \Rightarrow \boxed{\vec{r} \cdot \vec{c} = d}$ represent plane.

Ques 25) Explain covariant tensor of rank two.

Sol

Let A_{ij} ($i, j = 1, 2, \dots, n$) be n^2 fun. of the coordinates $x^1, x^2, \dots, x^n$. of the corresponding fun. $\bar{A}_{ij}$ of the co-ordinates $\bar{x}^1, \bar{x}^2, \dots, \bar{x}^n$ transform according to the law

$$A_{ij} = \frac{\partial x^k}{\partial \bar{x}^i} \frac{\partial x^l}{\partial \bar{x}^j} A_{kl}$$

then the fun. A_{ij} are called the components of a covariant tensor of the second order or rank two.

Ques 26 what is a geodesic on a surface.

Sol - A geodesic is a curve on a surface that represents the locally shortest path between two points.

Ex $\rightarrow$ on a sheet the geodesics are straight lines

Ques 27 If the metric is given by $ds^2 = 5(dx^1)^2 + 3(dx^2)^2 + 4(dx^3)^2 - 6dx^1dx^2 + 4dx^1dx^3$ then evaluate g . (the determinant of the metric tensor)

Sol - The general definition of a line element in a 3D space is given by

$$ds^2 = g_{11}(dx^1)^2 + g_{22}(dx^2)^2 + g_{33}(dx^3)^2 + 2g_{12}dx^1dx^2 + 2g_{13}dx^1dx^3 + 2g_{23}dx^2dx^3$$

compare it $g_{11}=5, g_{22}=3, g_{33}=4$

$$2g_{12} = -6 \Rightarrow g_{12} = g_{21} = -3, \quad 2g_{23} = 4 \Rightarrow g_{23} = g_{32} = 2$$

$$2g_{13} = 0 \Rightarrow g_{13} = g_{31} = 0$$

$$g_{ij} = \begin{pmatrix} 5 & -3 & 0 \\ -3 & 3 & 2 \\ 0 & 2 & 4 \end{pmatrix}$$

$$\det(g_{ij}) = 5 \cdot (12 - 4) + 3(-12 - 0) = 5(8) + 3(-12)$$

$$g = 40 - 36 \Rightarrow \boxed{g = 4}$$

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Ques 28 ~~Prove that~~ $\frac{dx^i}{dt} = v^i$ of a fluid at any point is a contravariant vector.

Sol The velocity of a fluid at any point has components $v^i = \frac{dx^i}{dt}$ in the co-ordinate system x^i . Also the components of velocity at the same point in another co-ordinate system $\bar{x}^i$ are $\bar{v}^i = \frac{d\bar{x}^i}{dt}$.

$$\frac{d\bar{x}^i}{dt} = \frac{\partial \bar{x}^i}{\partial x^1} \frac{dx^1}{dt} + \frac{\partial \bar{x}^i}{\partial x^2} \frac{dx^2}{dt} + \dots + \frac{\partial \bar{x}^i}{\partial x^n} \frac{dx^n}{dt}$$

$$\frac{d\bar{x}^i}{dt} = \frac{\partial \bar{x}^i}{\partial x^j} \frac{dx^j}{dt}$$

Ques 29 Prove that $\frac{\partial A_i}{\partial x^j}$ is not a tensor even though A_i is a covariant tensor of rank one.

Sol $A_i = \frac{\partial x^k}{\partial \bar{x}^i} A_k$ Differentiating both sides partially with respect to $\bar{x}^j$

$$\frac{\partial A_i}{\partial \bar{x}^j} = \frac{\partial x^k}{\partial \bar{x}^i} \frac{\partial A_k}{\partial \bar{x}^j} + \frac{\partial^2 x^k}{\partial \bar{x}^i \partial \bar{x}^j} A_k$$

$$= \left(\frac{\partial x^k}{\partial \bar{x}^i} \frac{\partial x^l}{\partial \bar{x}^j} \right) \frac{\partial A_k}{\partial x^l} + \frac{\partial^2 x^k}{\partial \bar{x}^i \partial \bar{x}^j} A_k$$

The presence of the second term on the right hand side implies that $\frac{\partial A_i}{\partial \bar{x}^j}$ do not transform as a tensor does.

Hence $\frac{\partial A_i}{\partial x^j}$ is not a tensor.

Ques 30 How many components does a tensor of rank 3 have in space of four dimensions.

Sol The no. of components in a tensor of rank n in n dimension is n^n i.e. $4^3 = 64$.