

Ques ① Define field with example.

Sol- A field is a commutative ring with unity in which every element (non-zero) has a multiplicative inverse. Example $\mathbb{R}, \mathbb{Q}, \mathbb{C}$

Field Axiom $\rightarrow$ (i) commutative group with respect to addition
(ii) " " " " " multiplication
(iii) Follow Distributive law.

Ques ② State Lagrange's theorem

Sol $\rightarrow$ If G is a finite group and H is a subgroup of G , then the order H divides the order of G . i.e. $|H| \mid |G|$
 $|H| \rightarrow$ represent the number of elements in subgroup H .
 $|G| \rightarrow$ " " " " " group G .

Ques ③ Find out the order of each element in the group $G = \{ \pm 1, \pm i \}$. (multiplication).

Sol:- $O(e) = 1 \Rightarrow O(1) = 1$ $O(i) = i \times i \times i \times i = 1 \Rightarrow O(i) = 4$
 $O(-1) = -1 \times -1 = 1 \Rightarrow O(-1) = 2$ $O(-i) = -i \times -i \times -i \times -i = 1 \Rightarrow O(-i) = 4$
 $\Rightarrow O(1) = 1, O(-1) = 2, O(i) = 4, O(-i) = 4.$

Ques ④ Show that the set of natural no. is not a group with respect to binary operation $+$.

Sol- set of Natural no. denoted as $\mathbb{N}$
 $\mathbb{N} = (\{ 1, 2, 3, \dots \}, +)$

For group four properties are must.

(i) closure (ii) Associative (iii) Identity (iv) Inverse

0 is the identity element in addition which doesn't belong to $\mathbb{N}$. Also inverse of each element doesn't exist in $\mathbb{N} \Rightarrow$ not a group.

Ques 5 Find Cayley table of the group $(G, +_4)$.

Sol For table

$+_4$	$[0]$	$[1]$	$[2]$	$[3]$
$[0]$	$[0]$	$[1]$	$[2]$	$[3]$
$[1]$	$[1]$	$[2]$	$[3]$	$[0]$
$[2]$	$[2]$	$[3]$	$[0]$	$[1]$
$[3]$	$[3]$	$[0]$	$[1]$	$[2]$

$G = \{[0], [1], [2], [3]\}$

Ques 6 If order of a cyclic group is 10. Find the order of subgroups.

Sol. Using Lagrange's theorem we know that order of a subgroup divides order of the group. So, 1, 2, 5, 10 are the only divisors.
2 trivial subgroups and 2 Non-trivial subgroups.

Ques 7 Define order of a group and order of the elements of the group with example.

Sol. Total Number of elements in G is called order of the group. It's denoted as $O(G)$ or $|G|$

Least integral power of each element which turns it into identity is called order of the element of the group.

Example - $G = \{ \pm 1, \pm i \} \Rightarrow O(G) = 4$

$$O(1) = 1, O(-1) = 2, O(i) = 4, O(-i) = 4.$$

Ques 8 If ϕ is Euler ϕ -function, then $\phi(10) = ?$

Sol Euler ϕ -fun. gives the no. of +ve integers upto n and relatively prime to n i.e. $\phi(n) = \phi(10) = \phi(2 \cdot 5) = \phi(2) \cdot \phi(5) = 4$

Ques 9 Define even & odd permutation with example ⁽²⁾

Sol - Even Permutation - An even permutation is a permutation that can be express as a product of an even number of transpositions (2-cycles)

Odd Permutation - If it can be express as a product of odd no. of transpositions.

Example - ① (1 2 3)

Decomposition (12)(13) even permutation.

② (1 2 3 4) decomposition (12)(13)(14) odd permutation

Ques 10 Test whether the cycle (1 2 3 4) is an even or odd Permutation.

Sol (1 2 3 4) Decomposition (12)(13)(14) odd permutation, $\therefore$ odd no. of transpositions

Ques 11 Show that the identity element of a group is unique.

Sol - suppose it's not unique. let e and e' two identity elements so. $e, e' \in G$

if we take e is identity then $ee' = e'$

Since e' is also identity then $e'e = e$

but in group $ee' = e'e$ so $\boxed{e' = e}$

Hence Proved

Ques 12 Evaluate the value of A^{-1} if $A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 1 & 5 & 4 \end{pmatrix}$

Sol - For $A^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 1 & 2 & 5 & 4 \end{pmatrix}$

Cross check $AA^{-1} = I$.

Ques (13) Prove that $(\mathbb{R}, +, \cdot)$ is a ring.

Sol- $(\mathbb{R}, +, \cdot)$ is a ring if it satisfies all below conditions

- (1) (i) Closure - sum of two reals is again real.
(ii) Associative - $(a+b)+c = a+(b+c) \quad \forall a, b, c \in \mathbb{R}$
(iii) Identity - $a+0 = 0+a = a \quad \forall a \in \mathbb{R}$.
(iv) Inverse - For each $a \in \mathbb{R} \exists -a \in \mathbb{R}$ such that $a+(-a)=0$
(v) Commutative - $a+b = b+a \quad \forall a, b \in \mathbb{R}$.
 $\Rightarrow (\mathbb{R}, +)$ is abelian group.

(2) $(\mathbb{R}, \cdot)$ is a semi-group.

- (i) closure - $a \cdot b \in \mathbb{R} \quad \forall a, b \in \mathbb{R}$
(ii) Associative $(a \cdot b) \cdot c = a \cdot (b \cdot c) \quad \forall a, b, c \in \mathbb{R}$.
 $\Rightarrow (\mathbb{R}, \cdot)$ is a semi group.

- (3) Distributive Law (i) $a \cdot (b+c) = a \cdot b + a \cdot c$ (Left distributive)
(ii) $(b+c) \cdot a = b \cdot a + c \cdot a$ (Right Distributive).

$\Rightarrow (\mathbb{R}, +, \cdot)$ is a ring.

Ques (14) Show that the relation R on Real numbers defined as $R = \{(a, b) : a \leq b\}$ is transitive.

Sol- To show that the relation $R = \{(a, b) : a \leq b\}$ on the set of real numbers $\mathbb{R}$ is transitive we have to show that
if $(a, b) \in R$ & $(b, c) \in R \Rightarrow (a, c) \in R$

$$\text{Now } (a, b) \in R \Rightarrow a \leq b \quad \text{--- (1)}$$

$$(b, c) \in R \Rightarrow b \leq c \quad \text{--- (2)}$$

Combine (1) & (2) together

$$a \leq b \leq c \Rightarrow a \leq c \Rightarrow (a, c) \in R.$$

Ques (15) Prove that in a group G $(ab)^{-1} = b^{-1}a^{-1} \quad \forall a, b \in G$

Sol- using definition of inverse we know that inverse is unique $\Rightarrow x^{-1} \in G \quad \forall x \in G$ is unique $\Rightarrow xx^{-1} = x^{-1}x = e$

now $(ab)(b^{-1}a^{-1}) = a(b \cdot b^{-1})a^{-1}$ (by associative) (3)
 $= a(e)a^{-1} = aa^{-1} = e$

$$(b^{-1}a^{-1})(ab) = b^{-1}(a^{-1}a)b = b^{-1}eb = b^{-1}b = e$$

$\Rightarrow$ inverse is unique so. $(ab)^{-1} = b^{-1}a^{-1}$
 $\forall a, b \in G$.

Ques 16 If $f = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$ and $g = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$ then find fg and gf .

Sol - $fg = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$ since $g(1)=1 \Rightarrow f(1)=3$ so $(fg)(1)=3$
 $g(2)=3 \Rightarrow f(3)=2$ so $(fg)(2)=2$

similarly $gf = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$

Ques 17 Show that the inverse of each element of a group is unique.

Sol - Let $(\text{identity})^{\text{same}}$ of gp is not unique and inverse of gp is not unique. (same process).

Here For inverse $a \in G$ and b, c two inverse of a in G .

$$\text{So } a \cdot b = b \cdot a = e \quad \text{--- (1)}$$

$$\text{and } a \cdot c = c \cdot a = e \quad \text{--- (2)}$$

$$\text{Now } b = b \cdot e = b \cdot (a \cdot c) = (b \cdot a) \cdot c = e \cdot c \Rightarrow \boxed{b=c}$$

using property of identity element, eq. (2), associative law. eq. (1)

Ques 18 Let G be a group, H is a subgroup of G . Let for $g \in G$, $gHg^{-1} = \{ghg^{-1} : h \in H\}$. Then prove that gHg^{-1} is a subgroup of G .

Sol - Here H is a subgroup $\Rightarrow e \in H$.

$$\text{Put } e \text{ on the set we get } geg^{-1} = gg^{-1} = e \Rightarrow e \in gHg^{-1}$$

$\Rightarrow gHg^{-1}$ is Non-empty

Let $x, y \in gHg^{-1} \Rightarrow x = gh_1g^{-1}$ & $y = gh_2g^{-1} \Rightarrow xy = gh_1(g^{-1}g)h_2g^{-1}$
 $xy = gh_1(e)h_2g^{-1} = g(h_1h_2)g^{-1} \Rightarrow$ closed under multiplication

Now $x \in gHg^{-1} \Rightarrow x = ghg^{-1}$ for some $h \in H \Rightarrow x^{-1} = (ghg^{-1})^{-1} \Rightarrow x^{-1} = (g^{-1})^{-1}h^{-1}g^{-1}$
 $\Rightarrow x^{-1} = gh^{-1}g^{-1}$ inverse closure holds $\Rightarrow gHg^{-1}$ is a subgroup.

Ques (19) Define (i) Fields (ii) Integral Domain.

Sol - Integral Domain - An integral domain is a commutative ring with a non-zero multiplicative identity that contains no-zero divisors.

Ex $\rightarrow$ Set of integers $\mathbb{Z}$ is an integral domain.

Field - A field is a commutative ring with a non-zero multiplicative identity ($1 \neq 0$) in which every non-zero element has a multiplicative inverse.

Ex - The set of rationals $\mathbb{Q}$ is a field.

Ques (20) Evaluate $L(\sin t \cos t)$

Sol - $L(\sin t \cos t) = \frac{1}{2} L\{2 \sin t \cos t\} = \frac{1}{2} L\{\sin 2t\}$

using $L\{\sin at\} = \frac{a}{p^2 + a^2} \quad b > 0$

we can write

$$\frac{1}{2} L(\sin 2t) = \frac{1}{2} \frac{2}{p^2 + 4} = \frac{1}{p^2 + 4}$$

Ques (21) Prove that $(x, y) \rightarrow (0, 0)$, $f(x, y) = \frac{xy^2}{x^2 + y^2}$ doesn't exist

Sol - Along x axis $y = 0$

$$f(x, 0) = \frac{x \cdot 0}{0 + x^2} = 0, \quad \lim_{x \rightarrow 0} f(x, 0) = 0$$

Along y axis $x = 0 \Rightarrow f(0, y) = \frac{0 \cdot y}{0 + y^2} = 0 \Rightarrow \lim_{y \rightarrow 0} \frac{0 \cdot y}{y^2 + 0} = 0$

Along $y = mx$

$$f(x, mx) = \frac{mx^3}{x^2 + m^2x^2} = \frac{m}{1 + m^2}$$

$\Rightarrow$ It depends upon path so take different values along different paths $\Rightarrow$ not exist.

Ques 22 Find $L(e^{-t} \sin^2 t)$

(4)

Sol - $\sin^2 t = \frac{1 - \cos 2t}{2}$, we know that $L\{e^{at} f(t)\} = F(s-a)$

$$L(e^{-t}) = \frac{1}{s+1} \quad \text{and} \quad L(e^{-t} \cos 2t) = \frac{s+1}{(s+1)^2 + 4}$$

$$\begin{aligned} \Rightarrow L(e^{-t} \sin^2 t) &= L\left(e^{-t} \left(\frac{1 - \cos 2t}{2}\right)\right) = \frac{1}{2} \left[\frac{1}{s+1} - \frac{s+1}{(s+1)^2 + 4} \right] \\ &= \frac{1}{2} \cdot \frac{4}{(s+1)\{(s+1)^2 + 4\}} = \frac{2}{(s+1)(s^2 + 2s + 5)} \end{aligned}$$

Ques 23 Show that $\lim_{(x,y) \rightarrow (0,0)} \frac{3x-2y}{2x-3y}$ doesn't exist.

Sol - Along any path $y = mx$ $\lim_{x \rightarrow 0} \frac{3x-2mx}{2x-3mx} = \frac{x(3-2m)}{x(2-3m)}$

$\Rightarrow$ It depends upon path $\Rightarrow$ doesn't exist.

Ques 24 Evaluate $L\{t \sinh at\}$.

$$\begin{aligned} \text{Sol - } L\{t \sinh at\} &= -\frac{d}{dp} L\{\sinh at\} = -\frac{d}{dp} \left(\frac{a}{p^2 - a^2} \right) \\ &= \frac{-2ap}{(p^2 - a^2)^2} = \frac{-2ap}{(p^2 - a^2)^2} \quad \text{here } p > |a| \end{aligned}$$

Ques 25 What is the meaning of a transposition.

Sol - A transposition in a permutation group is a cycle of length two. It swaps only any two elements and leaves all other elements unchanged.

Ex - $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 3 & 4 \end{pmatrix}$. or (12)

Ques 26 If kernel of Homomorphism is singleton then what does it mean.

Sol - A singleton kernel contains only the identity element of the domain. It means the homomorphism is injective.

i.e. No two distinct elements in the domain map to the same element in the co-domain. If $\phi(a) = \phi(b)$ then $a=b$.

Ques 27 what do you understand by Fourier Series?

Sol - If $f(x)$ is a periodic function with period $2l$ and satisfies Dirichlet's condition in the interval $(-l, l)$ then at every point of continuity, the Fourier series of $f(x)$ is given by the series

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left\{ a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right\}$$

$$a_0 = \frac{1}{l} \int_{-l}^l f(x) dx, \quad a_n = \frac{1}{l} \int_{-l}^l f(x) \cos \frac{n\pi x}{l} dx \quad \text{and}$$

$$b_n = \frac{1}{l} \int_{-l}^l f(x) \sin \frac{n\pi x}{l} dx, \quad n=1, 2, \dots$$

Here a_n, b_n are Fourier coefficients.

Ques 28 State Dirichlet's conditions.

Sol - A function $f(x)$ is said to satisfy Dirichlet condition in the interval (a, b) if

- (i) $f(x)$ is defined and single valued function except possibly at a finite number of points in the interval (a, b) , bdd also.
- (ii) $f(x)$ & $f'(x)$ are piecewise continuous in the interval (a, b) .
- (iii) Finite maxima and minima within the interval.

Ques (29) If $x = r \cos \theta$, $y = r \sin \theta$. Find $\frac{\partial(x,y)}{\partial(r,\theta)} = ?$ (5)

Sol $\rightarrow$ we know that $\frac{\partial(x,y)}{\partial(r,\theta)} = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{vmatrix} = \begin{vmatrix} \cos \theta & -r \sin \theta \\ \sin \theta & r \cos \theta \end{vmatrix}$

$$= r \cos^2 \theta + r \sin^2 \theta = r (\sin^2 \theta + \cos^2 \theta)$$
$$= r.$$

Ques (30) State fundamental theorem of Homomorphism of group.

Sol - Statement - If there is a group homomorphism from one group to another, the image of the homomorphism is isomorphic to the quotient group of the domain by the kernel.

Ques (31) Define Isomorphism with example.

Sol - A bijective mapping $f: G \rightarrow G'$ b/w two groups $(G, \cdot)$ and $(G', *)$ is called an isomorphism if it satisfies the homomorphism property for all $a, b \in G$

$$f(a \cdot b) = f(a) * f(b)$$

Ex A mapping $f: \mathbb{R} \rightarrow \mathbb{R}^+$ defined as $f(x) = e^x$.

Ques (32) Find all subgroups of the group $G = \{1, \omega, \omega^2\}$.

Sol - $O(G) = 3$ there doesn't exist any proper subgroup. because using Lagrange's theorem order of each subgroup divides order of the group. So there exist only non-trivial subgroups.

Ques (33) Define Normal subgroups of a group.

Sol - A subgroup N of a group G is said to be Normal subgroup if

$$N = \{ gng^{-1} \in N \mid n \in N, g \in G \}.$$

Ques 34 $L\{e^{2t}\} = ?$

Sol - we know that $L\{e^{at}\} = \frac{1}{p-a}$ $p > a$

$$\Rightarrow L\{e^{2t}\} = \frac{1}{p-2}$$

Ques 35 Define quotient group.

Sol - Let G be a group and H be its normal subgroup then

$$G/H = \{ha \mid a \in G\}$$

Here G/H is quotient group.

Ques 36 Define group and subgroup with example.

Sol - Group - A non empty set G together with binary operation satisfies following four properties

- (i) closure - $aob \in G \quad \forall a, b \in G$
- (ii) Associative - $aoboc = (aob)c \quad \forall a, b, c \in G$
- (iii) Identity - $aoc = coa = a; e \in G$ (iv) Inverse. $aoa^{-1} = e : a^{-1} \in G$

Subgroup $\rightarrow$ If we take non-empty subset H of G satisfying all above four properties, called subgroup of G .

Ex $\rightarrow (\mathbb{Z}, +)$ group $(2\mathbb{Z}, +)$ subgroup.

Ques 37 Show that every cyclic group is abelian.

Sol - let G be a cyclic group and let 'a' be a generator of G . Then $G = \langle a \rangle = \{a^n : n \in \mathbb{Z}\}$.

every element of G is of the form a^n for some integer n .

$$\text{Let } x, y \in G \Rightarrow x = a^r, y = a^s \Rightarrow x \cdot y = a^r \cdot a^s = a^{r+s}$$

$$y \cdot x = a^s \cdot a^r = a^{s+r}$$

Since $r+s = s+r$ in integers.

So. $x \cdot y = y \cdot x \Rightarrow$ commutative property holds trivially. So abelian.

Ques 38 Define Jacobian of n functions. (6)

Sol - Let $u_1, u_2, \dots, u_n$ be n functions of n independent variables $x_1, x_2, \dots, x_n$ i.e. $u_i = u_i(x_1, x_2, \dots, x_n)$ for $i=1, 2, \dots, n$, and all derivatives exist. Then determinant $n \times n$
$$\begin{vmatrix} \frac{\partial u_1}{\partial x_1} & \dots & \frac{\partial u_1}{\partial x_n} \\ \vdots & & \vdots \\ \frac{\partial u_n}{\partial x_1} & \dots & \frac{\partial u_n}{\partial x_n} \end{vmatrix}$$
 is called Jacobian of n fun.

Ques 39 Define continuity of fun. of two variable i.e. $f(x, y)$

Sol - A function $f(x, y)$ is said to be continuous at a point (a, b) if for every $\epsilon > 0$, $\exists \delta > 0$ such that

$$|f(x, y) - f(a, b)| < \epsilon \quad \text{where } |x - a| < \delta \text{ and } |y - b| < \delta$$

Ques 40 $L^{-1} \left\{ \frac{6}{s^4} \right\}$

Sol - $L\{t^n\} = \frac{n!}{s^{n+1}}$ Here $\frac{6}{s^4} = \frac{13}{s^{3+1}} \Rightarrow L^{-1} \left\{ \frac{6}{s^4} \right\} = t^3$

Ques 41 State First Shifting theorem in Laplace transform.

Sol - If $L\{f(t)\} = F(s)$ then $L\{e^{at}f(t)\} = F(s-a)$

$$L\{e^{at}f(t)\} = [F(s)]_{s \rightarrow s-a}$$

Ques 42 $L\{\sin at\}$

Sol $\rightarrow L\{\sin at\} = \int_0^\infty e^{-st} \sin at \, dt = \frac{a}{s^2 + a^2}$ For $a=1$

$$L\{\sin t\} = \frac{1}{s^2 + 1}$$

Ques 43 Define zero divisor with example.

Sol $\rightarrow$ Let R be a ring. A non-zero element $a \in R$ is called a zero divisor if $\exists$ a non-zero element $b \in R$ such that $ab = 0$ or $ba = 0$.

Ex $A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ $B = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow AB = 0$ neither $A=0$ nor $B=0$.

Ques 44 Define a division ring.

Sol - A ring R with unity $1 \neq 0$ is called a division ring or skewfield if every non-zero element has multiplicative inverse i.e. $\forall a \in R, a \neq 0 \exists b \in R : ab = ba = 1$. Ex $\rightarrow \mathbb{Q}, \mathbb{R}, \mathbb{C}$

Ques (45) Define limit point.

Sol - Let S be set and a pt. $p \in S$ is said to be a limit pt. of S if each nbd of p contains at least one pt. of S .

Ex $\rightarrow (a, b), [a, b), [a, b]$.

Ques (46) What is relation between Fourier and Laplace transform.

Sol - The Laplace Transformation is generalization of the Fourier transform.

Let $f(t)$ be a fun. for $t \geq 0$

Fourier transform $f(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt$. It exists only if $f(t)$ is absolutely integrable i.e. $\int_{-\infty}^{\infty} |f(t)| dt < \infty$

Laplace transform $F(s) = \int_0^{\infty} f(t) e^{-st} dt$ where s is complex $s = x + i\omega$

$$F(s) = \int_0^{\infty} f(t) e^{-(x+i\omega)t} dt = \int_0^{\infty} [f(t) e^{-xt}] e^{-i\omega t} dt$$

Ques (47) Define Fourier transformation.

Sol - Let $F(x)$ be a function defined on $(-\infty, \infty)$ and piecewise continuous in each finite partial interval and integrable in $(-\infty, \infty)$ then

$$F\{F(x)\} = \int_{-\infty}^{\infty} e^{-ipx} F(x) dx \text{ is called Fourier}$$

Transform of $F(x)$ denoted by $f(k)$.

Ques (48) State convolution theorem.

Sol - Statement - The Laplace transform of the convolution of two functions is equal to the product of their individual Laplace transforms.

$$L(f * g) = L(f) * L(g) = F(s) \cdot G(s)$$

defined as $f * g = \int_0^t f(\tau) g(t-\tau) d\tau$

Here $f(t)$ and $g(t)$ are two piecewise continuous functions on $[0, \infty)$ of exponential order. And their individual Laplace transforms are $L\{f(t)\} = F(s)$, $L\{g(t)\} = G(s)$.

Ques 49 Discuss the maxima or minima of the function

(7)

$$u = xy + \frac{a^3}{x} + \frac{a^3}{y}$$

Sol - $\frac{\partial u}{\partial x} = y - \frac{a^3}{x^2} = 0$ $\frac{\partial u}{\partial y} = x - \frac{a^3}{y^2} = 0$

Solve these equations we get $x^2y = a^3 = xy^2 \Rightarrow x = y = a$

for $x + s^2$ we do $r = \frac{\partial^2 u}{\partial x^2} = \frac{2a^3}{x^3} = 2$, $s = \frac{\partial^2 u}{\partial x \partial y} = 1$, $t = \frac{\partial^2 u}{\partial y^2} = \frac{2a^3}{y^3} = 2$

$x + s^2 = 2x^2 - 1^2 = 3 > 0$, by Lagrange's condition $x = y = a$

$$u_{\min} = aa + \frac{a^3}{a} + \frac{a^3}{a} = 3a^2$$

Ques 50 If $x = u(1+v)$ and $y = v(1+u)$ find the jacobian $\frac{\partial(x,y)}{\partial(u,v)} = ?$

Sol - $\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} 1+v & u \\ v & 1+u \end{vmatrix} = (1+v)(1+u) - uv = 1+u+v$

Ques 51 $L \{ t^2 e^{-at} \}$

Sol - We know that $L \{ t^2 F(t) \} = \frac{d^2}{dp^2} L \{ F(t) \}$.

$$L \{ t^2 e^{-at} \} = \frac{d^2}{dp^2} \left(\frac{1}{p+a} \right) = \frac{d}{dp} \left(\frac{-1}{(p+a)^2} \right) = \frac{2}{(p+a)^3}$$

Ques 52 $\int_0^\infty t e^{-3t} \sin t \, dt = \frac{3}{50}$

Sol - $L \{ t \sin t \} = -\frac{d}{ds} \left(\frac{1}{s^2+1} \right) = \frac{2s}{(s^2+1)^2}$

$$L \{ t \sin t \} = \int_0^\infty e^{-st} t \sin t \, dt = \frac{2s}{(s^2+1)^2}$$

Put $s=3$ on both side we get.

$$\int_0^\infty e^{-3t} t \sin t \, dt = \frac{2 \cdot 3}{(3^2+1)^2} = \frac{6}{(10)^2} = \frac{6}{100} = \frac{3}{50}$$

Ans

Ques 53 $\sum \frac{1}{n^2}$ cgt or not.

Sol using p-series test we know that $\sum \frac{1}{n^p}$ is cgt if $p > 1$
 $\Rightarrow \sum \frac{1}{n^2}$ is cgt.