

10.7 AMPERE'S CIRCUITAL LAW

Statement. The line integral $\oint \mathbf{B} \cdot d\mathbf{l}$ for a closed curve is equal to μ_0 times the net current i through the area bounded by the curve. That is,

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 i,$$

where μ_0 is the permeability constant.

Proof. Consider a long straight conductor carrying a current i perpendicular to the page directed outward (Fig. 10.15). According to Biot-Savart law, the magnitude of the magnetic induction at a distance r from it is given by

$$B = \frac{\mu_0 i}{2\pi r} \quad \dots (1)$$

At each point on this circle, B has a constant magnitude B and $d\mathbf{l}$ which is always tangential to the path of integration, points in the same direction as B . Thus,

$$\oint \mathbf{B} \cdot d\mathbf{l} = \oint B d\mathbf{l} = B \oint d\mathbf{l} = (B)(2\pi r)$$

Here, $2\pi r = \oint d\mathbf{l}$ is the circumference of the circle.

Substituting the value of B from Eq. (1), we get

$$\oint \mathbf{B} \cdot d\mathbf{l} = \frac{\mu_0 i}{2\pi r} (2\pi r) = \mu_0 i$$

Thus the integral $\oint \mathbf{B} \cdot d\mathbf{l}$ is μ_0 times the current through the area bounded by the circle. This is Ampere's law.

Differential Form of Ampere's Law

Let $\mathbf{j}$ be the current density in an element $d\mathbf{S}$ of the surface bounded by the closed path. Then,

$$\text{total current } i = \int_S \mathbf{j} \cdot d\mathbf{S}$$

$$\therefore \oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \int_S \mathbf{j} \cdot d\mathbf{S}$$

Using Stokes theorem,

$$\oint \mathbf{B} \cdot d\mathbf{l} = \int_S \text{curl } \mathbf{B} \cdot d\mathbf{S}$$

$$\therefore \int_S \text{curl } \mathbf{B} \cdot d\mathbf{S} = \mu_0 \int_S \mathbf{j} \cdot d\mathbf{S}$$

or

$$\text{curl } \mathbf{B} = \mu_0 \mathbf{j}$$

This is the differential form of Ampere's law.

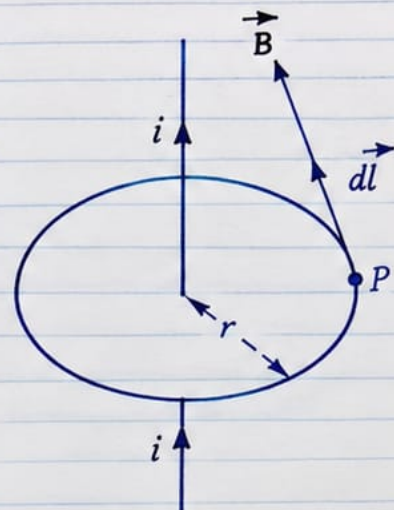


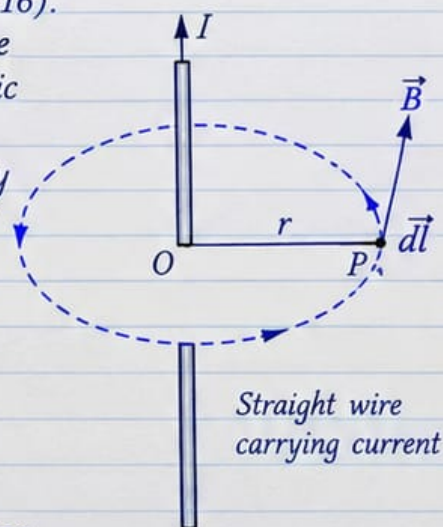
FIG. 10.15

10.8 APPLICATIONS OF AMPERE'S LAW

(i) Magnetic induction due to long straight current carrying wire:

Consider a long straight wire carrying a current I (Fig. 10.16). The magnetic lines of force are concentric circles centred on the wire. Let P be a point at distance r from the wire. The magnetic field at P is required.

Consider a circular path of radius r passing through P . By symmetry, the value of magnetic field B is same at each point on the circular path. Consider a small element dl of a line of magnetic field at P .



- B and dl are always directed along the same direction. The angle between B and dl is 0° .
- Line integral of B along the boundary of circular path,

$$\oint_C \mathbf{B} \cdot d\mathbf{l} = \int B dl \cos 0^\circ = B \oint_C dl = B 2\pi r.$$

From Ampere's theorem,

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \times \text{current enclosed by path}$$

$$B 2\pi r = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}.$$

(ii) Magnetic Field inside a long solenoid

Consider a long straight solenoid having n turns per unit length. Let i be the current flowing in the solenoid. It is experimentally noted that magnetic field outside the solenoid is very small in comparison with the field inside. The lines of induction inside the solenoid are straight and parallel (Fig. 10.17).

Consider a closed path $pqrs$. The line integral of magnetic field B along path $pqrs$ is

$$\oint_{pqrs} \mathbf{B} \cdot d\mathbf{l} = \int_{pq} \mathbf{B} \cdot d\mathbf{l} + \int_{qr} \mathbf{B} \cdot d\mathbf{l} + \int_{rs} \mathbf{B} \cdot d\mathbf{l} + \int_{sp} \mathbf{B} \cdot d\mathbf{l} \quad \dots(1)$$

Let $pq = l$. For path pq , B and dl are along same direction.

$$\therefore \int_{pq} \mathbf{B} \cdot d\mathbf{l} = \int B dl = Bl$$

For paths qr and sp , B and dl are mutually perpendicular.

$$\therefore \int_{qr} \mathbf{B} \cdot d\mathbf{l} = \int_{sp} \mathbf{B} \cdot d\mathbf{l} = \int B dl \cos 90^\circ = 0$$

For path rs , $B = 0$ (since field is zero outside a solenoid)

$$\therefore \int_{rs} \mathbf{B} \cdot d\mathbf{l} = 0$$

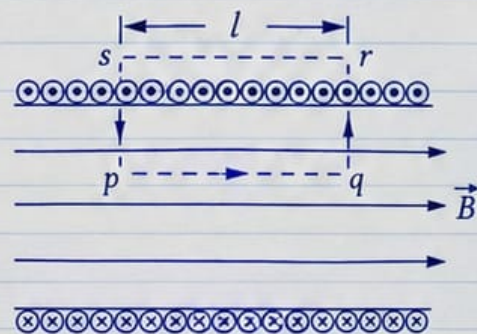


FIG. 10.17

$$\text{Eq. (1) becomes,} \quad \oint_{pqrs} \mathbf{B} \cdot d\mathbf{l} = \int_{pq} \mathbf{B} \cdot d\mathbf{l} = Bl \quad \dots(2)$$

By Ampere's law,

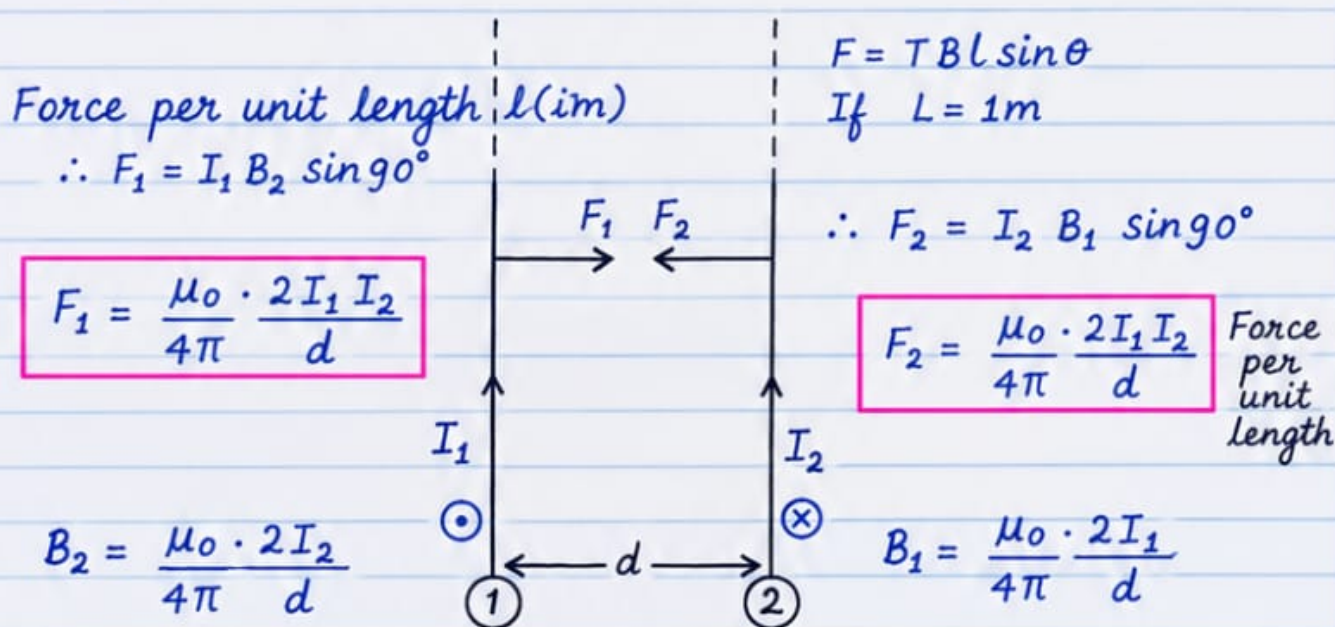
$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \times \text{net current enclosed by path}$$

$$Bl = \mu_0 (nl) i$$

$$B = \mu_0 n i \quad \dots(3)$$

• Force between two Parallel Current Carrying Conductors :

Consider two long parallel wires separated by a distance d' and carrying currents I_1 and I_2 respectively.



The wires attract each other if the currents in them are in the same direction and repel each other if the currents are in opposite directions.

• Numericals :

- ①. A current of 10 amp flows through each of the two parallel long wires which are 2cm apart. Calculate the force exerted per unit length of each

10

wire :

Solution. The force between two parallel wires each of length ' l ', carrying currents I_1 and I_2 and are separated by distance ' d ', is given by-

$$\frac{F}{l} = \frac{\mu_0 \cdot 2 I_1 I_2}{4\pi d}$$

Given : $I_1 = 10 \text{ amp}$, $I_2 = 10 \text{ amp}$, $d = 2 \text{ cm}$, $\frac{\mu_0}{4\pi} = 10^{-7} = 2 \times 10^{-2} \text{ m}$

$$\therefore \frac{F}{l} = \frac{10^{-7} \times 2 \times 10 \times 10}{2 \times 10^{-2}}$$

or,

$$\frac{F}{l} = 1 \times 10^{-3} \text{ N/m}$$

Ans

III Unit

Electromagnetic Induction

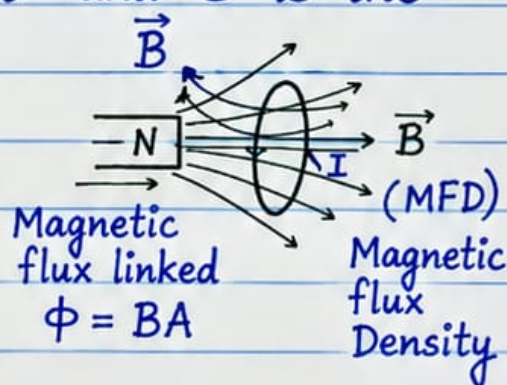
• Faraday's laws of Induction

Faraday gave two laws ;

- 1) Whenever the magnetic flux through a conductor is changed, an emf is induced in the conductor. This phenomenon is called electromagnetic induction.

The magnitude of the induced emf is equal to the rate of change of magnetic flux through the circuit. If Φ is the magnetic flux linked with the circuit at any instant 't' and $\mathcal{E}$ is the induced emf, then

$$\mathcal{E}(\text{Emf}) \propto \frac{d\Phi}{dt}$$



- 2) The direction of the induced emf or current, is such as to oppose the change that produced it. This is also Lenz's law.
known as

Combining both laws,

$$\mathcal{E} = - \frac{d\Phi}{dt}$$

Note :- 1) The production of induced emf or current in the coil is in accordance with the law of conservation of energy.

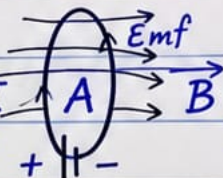
- 3) Lenz law gives : direction of induced emf or current.

$$\Phi \propto I$$

$$\Phi = LI$$

$$\Phi \propto B \propto I$$

Magnetic flux linked $\Phi = BA$



Where L is a constant, called the coefficient of self induction or self inductance of the coil.

If $I = 1$ Amp, then $L = \Phi$

Therefore, the self inductance of a coil is the total magnetic flux linked with it when a unit current passes through it.

When the flux changes, the back emf induced in the coil is given by (Faraday's law).

$$\mathcal{E} = -\frac{d\Phi}{dt} = -L \frac{dI}{dt}$$

Unit of self inductance is henry

$$= \frac{V \cdot s}{A} \quad \left(\frac{\text{volt} \cdot \text{second}}{\text{Ampere}} \right)$$

Dimension of L : $\left(L = \frac{\Phi}{I} \right)$

$$= \frac{\text{weber}}{\text{Ampere}} = \frac{ML^2 T^{-2} A^{-1}}{A}$$

$$= \frac{\text{weber}}{\text{Ampere}} = \frac{\text{Tesla} \cdot m^2}{\text{Ampere}}$$

$$D = ML^2 T^{-2} A^{-2}$$

- Self inductance of a long solenoid :-

11.2 FARADAY'S LAW OF ELECTROMAGNETIC INDUCTION IN VECTOR FORM

C is a closed circuit and S any surface with C as its boundary. Let B be the magnetic flux density in the neighbourhood of the circuit. The magnetic flux through a small area dS is $B \cdot d\vec{S}$.

The total magnetic flux linked with the circuit

$$\phi = \int_S B \cdot d\vec{S}. \quad \dots (1)$$

As the magnetic flux is changed, an electric field is induced around the circuit. The line integral of the electric field round the circuit gives the induced emf.

or

$$\varepsilon = \oint_C \vec{E} \cdot d\vec{l}. \quad \dots (2)$$

where $\vec{E}$ is the electric field at an element $d\vec{l}$ of the circuit.

According to Faraday's law, the emf induced in an electric circuit is equal to the negative of the time rate of change of the magnetic flux linked with the circuit.

or

$$\varepsilon = - \frac{d\phi}{dt} = - \frac{d}{dt} \int_S B \cdot d\vec{S}. \quad \dots (3)$$

From Eqns. (2) and (3),

$$\oint_C \vec{E} \cdot d\vec{l} = - \frac{d}{dt} \int_S B \cdot d\vec{S}. \quad \dots (4)$$

This is the integral form of the Faraday's law. It states that the line integral of the electric field around any closed circuit is equal to the negative time-rate of change of the magnetic flux linked with the circuit.

When $\vec{B}$ changes and circuit remains fixed, the time derivative can be taken inside the integral, where it becomes a partial time derivative.

$$\oint_C \vec{E} \cdot d\vec{l} = - \int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}. \quad \dots (5)$$

From Stokes' theorem

$$\oint_C \vec{E} \cdot d\vec{l} = \int_S \text{curl } \vec{E} \cdot d\vec{S}$$

Using this in equation(5),

$$\int_S \text{curl } \vec{E} \cdot d\vec{S} = - \int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}.$$

Therefore,

$$\text{curl } \vec{E} = - \frac{\partial \vec{B}}{\partial t} \quad \text{or} \quad \nabla \times \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

This is faraday's law of electromagnetic induction in differential form.

• Self-Inductance

The phenomenon of the production of an induced emf in a circuit itself due to the change in current through it is called self-induction and the induced emf is called back emf.

The magnetic flux ϕ produced in a coil is directly proportional to the current I following in the coil i.e.,

11.4 SELF-INDUCTANCE OF A LONG SOLENOID

Consider a long air-cored solenoid of length l , total number of turns N , and area of cross-section A . The number of turns per unit length is N/l . Let a current I flow through it. Then the magnetic field inside the solenoid is

$$B = \mu_0 \left(\frac{N}{l} \right) I$$

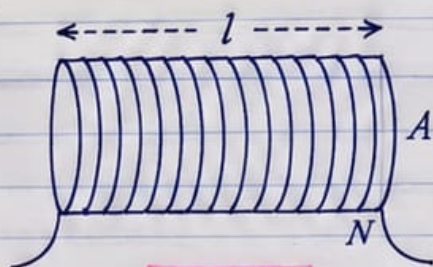


FIG. 11.6

$$\therefore \text{Flux through each turn} = \phi_B = BA = \frac{\mu_0 N I A}{l}$$

Total flux through N turns

$$\phi = N\phi_B = N \times \frac{\mu_0 N I A}{l} = \frac{\mu_0 N^2 I A}{l}$$

$$\therefore \left. \begin{array}{l} \text{self-inductance} \\ \text{of the solenoid} \end{array} \right\} L = \frac{\phi}{I} = \frac{\mu_0 N^2 A}{l}$$

If the solenoid is wound on a core of constant permeability μ , then

$$L = \frac{\mu N^2 A}{l}$$

where

$$\mu = \mu_0 \mu_r$$

In general, if there is a core consisting of a number of media of relative permeabilities, $\mu_{r1}, \mu_{r2} \dots$ etc., and areas of cross-section, $A_1, A_2, A_3 \dots$ etc, then,

$$L = \frac{\mu_0 N^2}{l} [\mu_{r1} A_1 + \mu_{r2} A_2 + \mu_{r3} A_3 + \dots]$$

Example 1. A solenoid having an air core and 10 cm long has 100 turns and area of cross-section is 5 sq. cm. Find the co-efficient of self-inductance of the solenoid.

Solution. Here, $l = 10 \text{ cm} = 0.1 \text{ m}$, $N = 100$, $A = 5 \text{ sq.cm} = 5 \times 10^{-4} \text{ m}^2$

$$\begin{aligned} \therefore L &= \frac{\mu_0 N^2 A}{l} \\ &= \frac{(4\pi \times 10^{-7}) (100 \times 100) (5 \times 10^{-4})}{0.1} = 62.8 \times 10^{-6} \text{ henry} \end{aligned}$$

Example 2. Calculate the self-inductance of a solenoid having 1000 turns and length 1 m. The area of cross-section is 7 cm² and the relative permeability of the core is 1000.

Solution. Here, $l = 1 \text{ m}$, $N = 1000$, $A = 7 \times 10^{-4} \text{ m}^2$, $\mu_r = 1000$

$$\begin{aligned} L &= \frac{\mu_r \mu_0 N^2 A}{l} = \frac{1000 \times (4\pi \times 10^{-7}) (1000)^2 (7 \times 10^{-4})}{1} \\ &= 0.88 \text{ henry} \end{aligned}$$

Example 3. A solenoid having a core of cross-section 4 cm², half air and half iron (relative permeability 500), is 22 cm long. If the number of turns on it is 1000, what will be its self-inductance?

Solution. Here, $N = 1000$, $l = 0.22 \text{ m}$, $A_1 = A_2 = 2 \times 10^{-4} \text{ m}^2$ and $\mu_r = 500$

$$\begin{aligned} \therefore L &= \frac{\mu_0 N^2 A_1}{l} + \frac{\mu_r \mu_0 N^2 A_2}{l} \\ &= \frac{(4\pi \times 10^{-7}) (1000)^2 (2 \times 10^{-4})}{0.22} + \frac{500 (4\pi \times 10^{-7}) (1000)^2 (2 \times 10^{-4})}{0.22} \\ &= 0.57 \text{ henry} \end{aligned}$$

- Energy stored in Magnetic field :

Consider an electric circuit containing inductance. When the circuit is closed, a back emf is induced in the circuit which opposes the growth of current in it. Therefore a certain amount of work has to be done by the current in increasing the current from zero to a maximum value against the induced back emf. The work done by the current is

stored in the magnetic field of the coil as potential energy.

Let I be the current in the inductor L at any instant t . Let the rate of growth of current be $\frac{dI}{dt}$.

The back emf induced in the inductor,

$$\mathcal{E} = -L \frac{dI}{dt}$$

The work done in moving charge dq against this emf is $dw = -\mathcal{E} dq$

$$dw = L \frac{dI}{dt} dq$$

$$dw = L \frac{dq}{dt} dI$$

$$dw = LI dI$$

Total work done in increasing the current from zero to maximum value I_0 is

$$W = \int_0^{I_0} LI dI$$

$$W = L \left[\frac{I^2}{2} \right]_0^{I_0}$$

$$W = \frac{1}{2} L I_0^2$$

Thus, the energy stored in the inductor is

$$U = \frac{1}{2} L I_0^2$$

Consider a very long air cored solenoid of length l , cross-sectional area A , and total no. of turns N . Let I be the current in solenoid.
Magnetic induction along axis inside the solenoid,

$$B = \mu_0 \left(\frac{N}{l} \right) I$$

or,
$$I = \frac{Bl}{\mu_0 N}$$

The self-inductance of a solenoid,

$$L = \frac{\mu_0 N^2 A}{l}$$

Therefore, energy stored in the magnetic field is

$$W = \frac{1}{2} \frac{\mu_0 N^2 A}{l} \frac{B^2 l^2}{\mu_0^2 N^2}$$

$$W = \frac{1}{2} \frac{B^2 A l}{\mu_0}$$

Then,

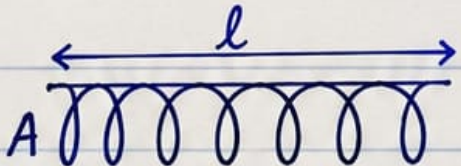
$$U = \frac{B^2 A l}{2 \mu_0}$$

Volume of solenoid = Al

Therefore, energy stored per unit volume or the energy density ' u ' in the magnetic field is

$$u = \frac{U}{Al} = \frac{B^2}{2 \mu_0}$$

Q → A coil of radius r has 600 turns and a self-inductance of 108 millihenry (mH). The self-inductance of similar coil of 500 turns will be -

Given:  $L = 108 \text{ mH}$, $N = 600$
 $N' = 500$, $L' = ?$

The self-inductance of a coil,

$$L = \mu_0 \frac{N^2 A}{l}$$

$$108 \text{ mH} = \frac{\mu_0 (600)^2 A}{l} \longrightarrow \textcircled{1}$$

Again,

$$L' = \frac{\mu_0 (500)^2 A}{l} \longrightarrow \textcircled{2}$$

From eq $\textcircled{1}$ and $\textcircled{2}$,

$$\therefore \frac{L'}{L} \Rightarrow \frac{L'}{108 \text{ mH}} = \frac{(500)^2}{(600)^2} = \frac{25}{36}$$

$$L' = \frac{25}{36} \times 108 \text{ mH}$$

$$L' = 25 \times 3 \text{ mH}$$

$$L' = \underline{\underline{75 \text{ mH}}}$$